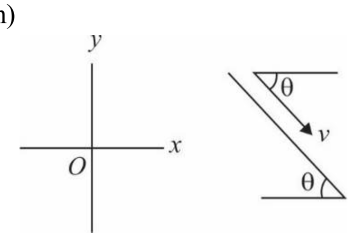
# Solutions of JEE Advanced-1 | Paper–2 | JEE 2024

## PHYSICS

### SECTION - 1 | SINGLE CHOICE CORRECT TYPE

**1.(B)** $\vec{v}_m = v\cos\theta\hat{i} - v\sin\theta\hat{j} = \frac{6}{2}\hat{i} - \frac{6\sqrt{3}}{2}\hat{j} = 3i - 3\sqrt{3}\hat{j}$ (velocity of man)




$\vec{v}_R = -v'\hat{j}$ (velocity of rain)

$\vec{v}_{Rm} = \vec{v}_R - \vec{v}_m = (-v' + 3\sqrt{3})\hat{j} - 3\hat{i}$ (velocity of rain w.r.t. man)

Since $\vec{v}_{Rm}$ is horizontal, $v_y = 0 = -v' + 3\sqrt{3}$

$\Rightarrow \quad v' = 3\sqrt{3}\ \text{ms}^{-1}$ hence $|\vec{v}_R| = 3\sqrt{3}\ \text{ms}^{-1}$

**2.(C)** Time taken by stone $A$ to reach its highest point, $T = \frac{v_0}{g}$

Since the distance travelled by stone $B$ during time $T$ is $h$,

$$v_0 T + \frac{1}{2}gT^2 = h \quad \Rightarrow \quad v_0\left(\frac{v_0}{g}\right) + \frac{1}{2}g\left(\frac{v_0}{g}\right)^2 = h \quad \Rightarrow \quad v_0 = \sqrt{\frac{2gh}{3}}$$

**3.(D)** Since $B$ moves faster than $A$, clearly it will cover a greater distance before they meet

So, we can look at the situation as $B$ being three-quarters of the circle, i.e. a distance $\frac{3\pi R}{2}$ behind $A$ initially. Hence, the time instant when they meet is given by

$$\left(\frac{3v}{2}\right)t = vt + \frac{3\pi R}{2} \quad \Rightarrow \quad t = \frac{3\pi R}{v}$$

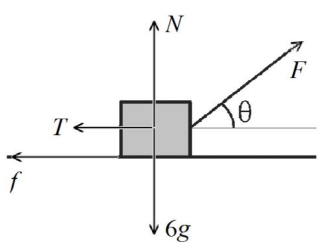
**4.(D)** From the FBD of $B$, $T = 1g = 10\ N$

Now, let friction act leftwards on $A$

Assuming equilibrium,

$$N + F\sin\theta = 6g \quad \text{and} \quad T + f = F\cos\theta$$

$$\Rightarrow \quad N = 60 - 0.6F \quad \text{and} \quad f = 0.8F - 10$$




We know that the magnitude of friction must be less than or equal to $\mu N$. But, since the friction can act rightwards as well (which can simply be represented in our equations by replacing $f$ by $-f$), the complete inequality on $f$ is

$$-\mu N \le f \le \mu N$$

$$\Rightarrow \quad -(0.1)(60 - 0.6F) \le (0.8F - 10) \le (0.1)(60 - 0.6F)$$

$\Rightarrow \quad -(6-0.06F) \le (0.8F-10) \le (6-0.06F)$

$\Rightarrow \quad (0.8F-10) \ge -(6-0.06F)$ and $(0.8F-10) \le (6-0.06F)$

$\Rightarrow \quad 0.74F \ge 4$ and $0.86F \le 16$

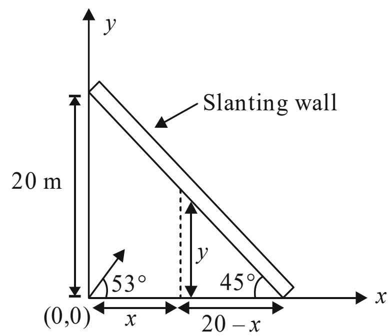
$\Rightarrow \quad F \ge 5.41$ and $F \le 18.61$

So, the complete condition on $F$ is $5.41N \le F \le 18.61\ N$

**SECTION 2 | MULTIPLE CORRECT ANSWERS TYPE**

**5.(BD)** Equation of the wall:

$$y = 20 - x \qquad \dots (1)$$




Equation of trajectory of the projectile:

$$y = x\tan 53° - \frac{10\times x^2}{2\times 250}\times(\sec^2 53°)$$

$$y = \frac{4}{3}x - \frac{x^2}{50}\times\frac{25}{9}$$

$$y = \frac{4}{3}x - \frac{x^2}{18};\quad 20 - x = \frac{4}{3}x - \frac{x^2}{18}$$

$$x^2 - 18x - 24x + 360 = 0$$

$$x^2 - 42x + 360 = 0 \quad \Rightarrow \quad x = 12m, 30m$$

Valid $x$ = 12 m $\Rightarrow$ $y$ = 8 m (from (1)) $\therefore$ Coordinates are (12, 8)

Its velocity when it hits the wall

$$u_x = 5\sqrt{10}\cos 30° = 3\sqrt{10} \text{ m/s}$$

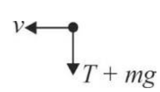
$x$ = 12 m at time collision $t = \dfrac{12}{3\sqrt{10}} = \dfrac{4}{\sqrt{10}} s$

$$u_y = 5\sqrt{10}\times\sin 53° = 4\sqrt{10} \text{ m/s} \quad ; \quad v_y = 4\sqrt{10} - 10\times\frac{4}{\sqrt{10}} = 0$$

Therefore, the velocity of the projectile when it hits the wall, $\vec{v} = 3\sqrt{10}\,\hat{i}$ m/s

**6.(BCD)** At $C: T + mg = \dfrac{mv^2}{\ell}$

$\Rightarrow \quad mg + mg = \dfrac{mv^2}{\ell} \Rightarrow v = \sqrt{2g\ell}$




Applying energy conservation from $A$ to $C$

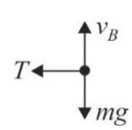
$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg(2\ell)$$

$\Rightarrow \quad u^2 = v^2 + 4g\ell = 2g\ell + 4g\ell \quad \therefore \quad u = \sqrt{6g\ell}$

Velocity at $B$ can be found by applying energy conservation from $A$ to $B$

$$\frac{1}{2}mu^2 = \frac{1}{2}mv_B^2 + mg\ell$$

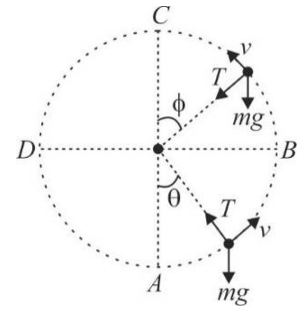
$$v_B^2 = u^2 - 2g\ell = 6g\ell - 2g\ell$$



$\therefore \quad v_B = 2\sqrt{g\ell}$

Tension provides the required centripetal acceleration

$$\therefore \quad T = \frac{mv_B^2}{\ell} = \frac{m(4g\ell)}{\ell} = 4mg$$




As bob moves up, its potential energy increases and kinetic energy decreases.

So, speed decreases continuously while going from $A$ to $B$ to $C$

$$A-B:\ T - mg\cos\theta = \frac{mv^2}{\ell}; \qquad T = mg\cos\theta + \frac{mv^2}{\ell}$$

As $\theta$ increases, $\cos\theta$ decreases. As both terms on RHS decreases. So $T$ decrease

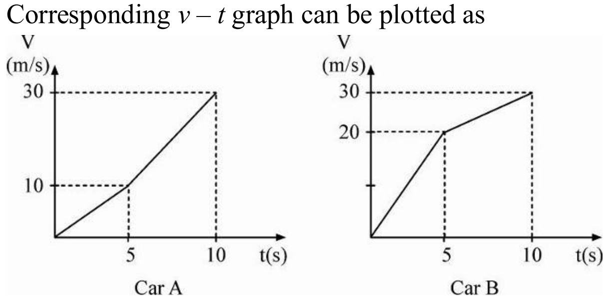
$$B-C:\ mg\cos\phi + T = \frac{mv^2}{\ell}; \qquad T = \frac{mv^2}{\ell} - mg\cos\phi$$

As $\phi$ decreases, $\cos\phi$ increases.

As first term in RHS decreases and second term increases. So $T$ decreases

**7.(AC)** Since area under $a-t$ graph is same for both, so change in velocity is same.

Corresponding $v-t$ graph can be plotted as




Since area under $v-t$ graph is more for car $B$, so it travels more distance

**8.(BC)** $\dfrac{x}{v+u} = t_1 \quad \Rightarrow \quad v+u = \dfrac{x}{t_1}$

$$\frac{x}{v-u} = t_2 \quad \Rightarrow \quad v-u = \frac{x}{t_2}$$

$$v = \frac{x}{2}\left(\frac{1}{t_1} + \frac{1}{t_2}\right); \qquad u = \frac{x}{2}\left(\frac{1}{t_1} - \frac{1}{t_2}\right)$$

**9.(BD)** Velocity of block is maximum when $mg = kx \quad \Rightarrow \quad x = \dfrac{10}{100} = 0.1m$

Compression is maximum when velocity of block is zero.

Applying energy conservation

$$mg(h+x) = \frac{1}{2}kx^2 \quad \Rightarrow \quad 10(0.4+x) = 50x^2$$

$$\Rightarrow \quad 5x^2 - x - 0.4 = 0; \qquad x = \frac{1 \pm \sqrt{1+8}}{10} = \frac{1+3}{10} = 0.4m$$

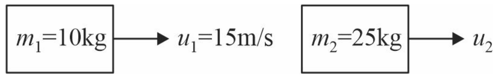
**10.(ABC)** From graph in time from $t=0$ to $t=3$ sec. acceleration of object of mass $m_1 = 10$ kg is

$a = \dfrac{15-0}{3} = 5 \text{ m/s}^2$

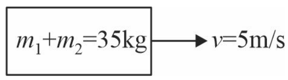
$\therefore$ Force on object of mass $m_1$ from $t=0$ to $t=3$ sec. … (i)

$= 10 \times 5 = 50N$

Before and after collision at $t = 5$ sec, the velocities of blocks are as shown.




Before collision




After collision

$\therefore$ Initial momentum of system $= m_1u_1 + m_2u_2 = 150 + 25u_2$

Final momentum of system $= (m_1 + m_2)v = 35 \times 5 = 175$

From conservation of momentum

$\therefore$ $150 + 25u_2 = 175$ or $u_2 = +1$ m/s

$\therefore$ speed of second particle just before collision is 1 m/s and before collision both blocks move in same direction.

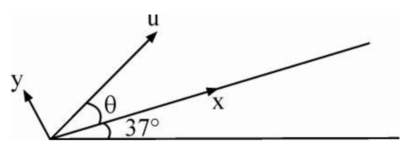
## SECTION 3 | SINGLE DIGIT INTEGER TYPE

**1.(1)** Taking $x$ and $y$ as shown, for motion of ball :

$u_x = u\cos\theta + 5, u_y = u\sin\theta$

$a_x = -g\sin 37° = -6$

$a_y = -g\cos 37° = -8$




At the centre at hoop, $S_y = 4$ and $V_y = 0$

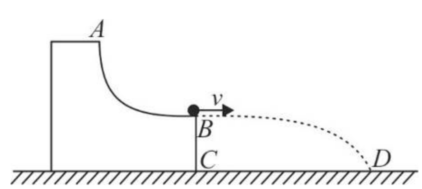
$\therefore$ $V_y^2 = u_y^2 + 2a_ys_y$ $\Rightarrow$ $0 = (u\sin\theta)^2 + 2(-8)(4)$

$\Rightarrow$ $u\sin\theta = 8$

Also $V_y = u_y + a_yt$ $\Rightarrow$ $0 = u\sin\theta + (-8)t$ $\Rightarrow$ $t = \dfrac{u\sin\theta}{8} = \dfrac{8}{8} = 1$

**2.(8)** Velocity with which it leaves the track can be found by energy conservation.

$\dfrac{1}{2}mv^2 = mgR$ $\Rightarrow$ $v = \sqrt{2gR}$




After that, it undergoes a projectile motion till it hits the ground.

Time of flight will be

$T = \sqrt{\dfrac{2h}{g}}$; $CD$ = range $= vT = 2\sqrt{Rh} = 8m$

**3.(8)** Work done = Change in kinetic energy

$4(8) = \dfrac{1}{2}(1)v^2$ $\Rightarrow$ $v = 8$ m/s

**4.(8)** Let the acceleration of the blocks be $a_1$, $a_2$ and $a_3$ respectively, all assumed upwards

Let the mass of $A$ be $m$

Then, $2T - mg = ma_1$

$T - 3g = 3a_2$; $T - 2g = 2a_3$

Therefore, $a_1 = \frac{2T}{m} - g$; $a_2 = \frac{T}{3} - g$; $a_3 = \frac{T}{2} - g$

But we know that $a_2 + a_3 = -2a_1$

$\Rightarrow \left(\frac{T}{3} - g\right) + \left(\frac{T}{2} - g\right) = -2\left(\frac{2T}{m} - g\right) \Rightarrow T = \frac{4g}{\frac{5}{6} + \frac{4}{m}} = \frac{24mg}{(5m+24)}$

But, for the block $B$ to accelerate upwards, $a_2 > 0$

$\Rightarrow \frac{T}{3} - g > 0 \Rightarrow T > 3g$

$\Rightarrow \frac{24mg}{(5m+24)} > 3g \Rightarrow m > 8$

**Alternative solution**

It is quite intuitive that if the block $A$ is heavier, the block $B$ will accelerate upwards, and if $A$ is lighter, $B$ will accelerate downwards. Thus, there exists a value of the mass of $A$ for which $B$ remains at rest (while $A$ and $C$ accelerate), and this is the value that we need to find. We can find it by replacing $a_2 = 0$ in the equations.

**5.(8)** Maximum speed, $v_M = 6(5) = 30$ m/s

Therefore, $30 + (-2)(T-5) = 0 \Rightarrow T = 20$ s

So, the total time for which the car moved is 20 s.

Distance travelled between $t = 0$ and $t = 5$,

$$D_1 = \frac{1}{2}(6)(6)^2 = 75 \text{ m}$$

Distance travelled between $t = 5$ and $t = 20$,

$$D_2 = 30(15) + \frac{1}{2}(-2)(15)^2 = 225 \text{ m}$$

Since $D_1 < D_2$, we can be sure that the instant $t = T_0$ lies between $t = 5$ and $t = 20$

So, equating the distance travelled between $t = 0$ and $t = T_0$ to the distance travelled between $t = T_0$ and $t = T$,

$$75 + 30(T_0 - 5) + \frac{1}{2}(-2)(T_0 - 5)^2 = (30 + (-2)(T_0 - 5))(20 - T_0) + \frac{1}{2}(-2)(20 - T_0)^2$$

$\Rightarrow 30T_0 - 75 - (T_0 - 5)^2 = 2(20 - T_0)^2 - (20 - T_0)^2$

$\Rightarrow 30T_0 - 75 = (20 - T_0)^2 + (T_0 - 5)^2$

$\Rightarrow T_0^2 - 40T_0 + 250 = 0 \Rightarrow T_0 = 20 - 5\sqrt{6}$

So, to the nearest integer, $T_0 = 8$ s

**6.(4)** At maximum elongation $(x_{\max})$ of spring, speed of $m$ is zero and it moves down by $2x_{\max}$

Applying energy conservation:

$$-mg2x_{\max} - \frac{1}{2}K[(x_{\max})^2 - 0^2] = 0 \quad \Rightarrow \quad x_{\max} = \frac{4mg}{k}$$

**7.(2)** They can avoid the collision when separation between them starts.

First $A$ throws ball towards $B$. Applying conservation of momentum on '$A$ + ball' system

$$80.1 = 70V_A + 10(5 + V_A)$$

Where $V_A$ is speed of $A$ towards $B$ after throwing the ball.

$$V_A = \frac{3}{8} \text{ m/s}$$

$B$ catches the ball and throws towards $A$. Let $V_B$ is speed of $B$ towards $A$ after the throw. Therefore

$$70.1 - 10.\frac{43}{8} = 70.V_B + 10(5 + V_B)$$

$$\frac{130}{8} - 50 = 80V_B$$

$$-\frac{370}{640} = V_B \quad \Rightarrow \quad V_B = -\frac{1}{2}$$

i.e. $B$ is going towards right with speed more than that of $A$ (they are separating).

**8.(5)** The sliding shall start at lower surface first

It $\quad F > 0.5[10+10]g \quad$ or $\quad F > 100N$

# CHEMISTRY

## SECTION - 1 | SINGLE CHOICE CORRECT TYPE

**1.(A)** Let empirical formula of the compound is $C_xH_yO_z$.

$$C_xH_yO_z + \left(x + \frac{y}{4} - \frac{z}{2}\right)O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$$

Volume of $O_2$ required is, $(x + \frac{y}{4} - \frac{z}{2}) = 2.5$ …… (1)

Volume of $CO_2$ formed is, $x = 2$ …… (2)

Volume of water vapour formed is, $\frac{y}{2} = 2$ …… (3)

$x = 2$, $y = 4$ and $z = 1$

**2.(A)** According to VSEPR theory electronegativity of surrounding atom also affect B – A – B bond angle in $AB_n$ type species. F is more electronegative than N hence repulsion between bond pairs is less in $NF_3$ than in $NH_3$.

**3.(A)** $$\frac{1}{300} = \frac{1}{760} + \frac{1}{x}$$

$$\frac{1}{300} - \frac{1}{760} = \frac{1}{x}$$

$$\frac{760-300}{760\times300} = \frac{460}{760\times300} = \frac{1}{x}, x = 495.65 \cong 496\text{nm}$$

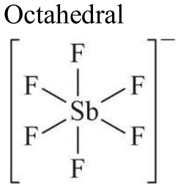
**4.(C)** In low pressure region value of $Z = 1$.

## SECTION 2 | MULTIPLE CORRECT ANSWERS TYPE

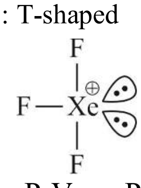
**5.(BD)** For homonuclear diatomic molecular species having 14 or a smaller number of electrons, bonding $\sigma(2p)$ orbital is higher in energy than $\pi(2p)$ orbitals.

**6.(ABD)**

$SbF_6^-$ : Octahedral




$XeF_3^+$ : T-shaped




**7.(CD)** $$\frac{P_0V_0}{RT_0} + \frac{P_0V_0}{RT_0} = \frac{PV_0}{RT_0} + \frac{PV_0}{R.3T_0}$$

$$\frac{2P_0}{T_0} = \frac{4P}{3T_0} \Rightarrow P = \frac{3}{2}P_0$$

$$n = \frac{P\cdot V_0}{R\cdot 3T_0} = \frac{3}{2}\frac{P_0V_0}{R\cdot 3T_0} = \frac{P_0V_0}{2RT_0}$$

**8.(BCD)**

$$\begin{array}{llll} 2NO(g) & + \; O_2(g) & \longrightarrow & 2NO_2(g) \\ 2\text{ mole} & 1\text{ mole} & & 2\text{ mole} \rightarrow \text{heat released} = 35\text{ kcal} \\ 6\text{ mole} & 3\text{ mole} & & 6\text{ mole} \rightarrow \text{heat released} = 105\text{ kcal} \end{array}$$

$\Rightarrow \quad 6NO(g) + 3O_2(g) \longrightarrow 6NO_2(g)$

$$\Delta H = \Delta U + \Delta n_g \cdot RT$$

$$-105 = \Delta U + (-3) \times \frac{2 \times 300}{1000}$$

$$\Delta U = -105 + 1.8 = -103.2\,\text{kcal} \; ; \; q = \Delta U - w$$

$$-105 = -103.2 - w \quad \Rightarrow \quad w = 105 - 103.2 = 1.8\,\text{kcal}$$

**9.(ABD)**

Theory based

**10.(BD)**

At point a,

$$P_0V_0 = 1 \times RT_0 \quad \Rightarrow \quad T_0 = \frac{P_0V_0}{R}$$

At point b,

$$P_0 \times 4V_0 = 1 \times R \times T_b \quad \Rightarrow \quad T_b = \frac{4P_0V_0}{R} = 4T_0$$

At point c,

$$2P_0 \times 4V_0 = 1 \times R \times T_c \quad \Rightarrow \quad T_c = \frac{8P_0V_0}{R} = 8T_0$$

$$\Delta U_{a \to b} = 1 \times \frac{3}{2} R \times (4T_0 - T_0) = \frac{9}{2} RT_0$$

$$\Delta U_{b \to c} = 1 \times \frac{3}{2} R \times (8T_0 - 4T_0) = 6\, RT_0$$

$$\Delta U_{a \to c} = \Delta U_{a \to b} + \Delta U_{b \to c} = \frac{9}{2} RT_0 + 6RT_0$$

$$\Delta U_{a \to c} = \frac{21}{2} RT_0$$

$$W_{a \to b} = -P_0 \times (4V_0 - V_0) = -3P_0V_0 = -3RT_0$$

$$\Delta U_{a \to b} = q_{a \to b} + W_{a \to b}$$

$$\frac{9}{2} RT_0 = q_{a \to b} - 3RT_0$$

## SECTION 3 | SINGLE DIGIT INTEGER TYPE

**1.(6)** T = constant

$$P_1V_1 = P_2V_2$$

$$7 \times 1 = \left(\frac{760}{760}\right) \times V_2 \;\Rightarrow\; V_2 = 7\,\text{litre}$$

$$W = -P(\Delta V) = -1 \times (7-1) = -6\,\text{Latm}$$

**2.(6)** Electronic configuration of As, $1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 4s^2, 3d^{10}, 4p^3$. Electrons of 3p have n = 3 and $l$ = 1.

**3.(3)** Reaction for formation of complex

$$Cu^{2+}(aq) + 4NH_3(aq) \rightleftharpoons [Cu(NH_3)_4]^{2+}(aq)$$

$$K_f = \frac{[Cu(NH_3)_4]^{2+}}{[Cu^{2+}] \times [NH_3]^4} = \frac{(0.99)}{[NH_3]^4 \times 10^{-2}}$$

$$[NH_3]^4 = \frac{0.99}{1.1 \times 10^{11}}$$

$$[NH_3] = 0.001732 = 1.732 \times 10^{-3}$$

**4.(3)** The change in sign of radial wavefunction indicate the presence of nodes.

Number of radial nodes = (n – $l$ – 1)

For this orbital, number of radial nodes are 2 and it is ns orbital because $\Psi > 0$ at r = 0.

2 = (n – 0 – 1) $\Rightarrow$ n = 3

**5.(3)** Let oxidation state of metal ion in the metal bromide be n.

$$MBr_n + nAgNO_3 \longrightarrow M(NO_3)_n + nAgBr$$

Mole of $AgNO_3$ used = n × Mole of $MBr_n$

$$0.025 \times \frac{60}{1000} = n \times 0.0005 \quad \Rightarrow \quad n = \frac{0.025 \times 60}{1000 \times 0.0005} = 3$$

**6.(3)** $2e^- + 2H^+ + H_3AsO_4 \longrightarrow H_3AsO_3 + H_2O$

$$2I^- \longrightarrow I_2 + 2e^-$$

$$\text{moles of } e^- = \frac{1.5 \times 10^{22}}{6 \times 10^{23}} = \frac{1}{40}\text{ mole}$$

$$\text{moles of } I_2 = \frac{1}{80}\text{ mol}$$

$$\text{mass of } I_2 = \frac{1}{80} \times 254 = 3.175\,\text{gm}$$

**7.(2)** $Cu + 4HNO_3 \longrightarrow Cu(NO_3)_2 + 2NO_2 + 2H_2O$

a = 1, b = 4, c = 1, d = 2, e = 2

**8.(4)** $n - 5 = 1 \Rightarrow n = 6$

Electronic configuration of element is:

$$1s^2,\ 2s^2\,2p^6,\ 3s^2\,3p^6,\ 4s^2\,3d^{10}\,4p^6,\ 5s^2\,4d^{10}\,5p^6,\ 6s^2\,4f^{10}$$

Number of unpaired $e^-$ = | ⇅ | ⇅ | ⇅ | ↑ | ↑ | ↑ | ↑ | = 4

# MATHEMATICS

### SECTION - 1 | SINGLE CHOICE CORRECT TYPE

**1.(D)** **Case – I :** Let $x \le 0$ then $2.3^x \le 2$ and $\dfrac{4x^2+x+2}{x^2+x+1} \ge 2 \Rightarrow x \le 0$ or $x \ge \dfrac{1}{2}$. So, $x \le 0$

**Case –**II: Let $x > 0,$ we prove that $\dfrac{4x^2+x+2}{x^2+x+1} < 2.3^x$

Assume the opposite i.e. $\dfrac{4x^2+x+2}{x^2+x+1} \ge 2.3^x$

$\because \quad \dfrac{4x^2+x+2}{x^2+x+1} > 2.3^{\circ} = 2 \Rightarrow x < 0$ or $x > \dfrac{1}{2}$

Since, $x > 0$. So, $x > \dfrac{1}{2}$. Hence, $\dfrac{4x^2+x+2}{x^2+x+1} \ge 2.3^x > 2\sqrt{3} > 3$

$\Rightarrow \quad x^2 - 2x - 1 > 0 \Rightarrow x \in \left(1+\sqrt{2}, \infty\right)$

Thus, $\dfrac{4x^2+x+2}{x^2+x+1} \ge 2.3^x > 23^{1+\sqrt{2}} > 2.3^2 = 18$

But $\dfrac{4x^2+x+2}{x^2+x+1} < 4$ for any $x > 0,$ we get a contradiction

So, domain is $R$

**2.(C)** $ar^5 = 4(ar^3) \;\Rightarrow\; r^2 = 4 \;\Rightarrow\; r = 2$

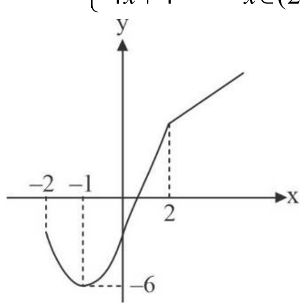
$a2^8 - a(2^6) = 192 \;\Rightarrow\; a = 1$

$S_n - S_3 = 1016$

$(2^n - 1) - (1+2+4) = 1016$

$2^n = 1024 \;\Rightarrow\; n = 10$

**3.(C)** $f(x) = \begin{cases} 2x^2+4x-4 & x \in [-2, 2] \\ 4x+4 & x \in (2, \infty) \end{cases}$




$\therefore \quad$ Range : $[-6, \infty)$

**4.(B)** $f(-4) = a(-4)^4 + b(-4)^2 - 12 + 7 = 2286$

$f(4) = a(4)^4 + b(4)^2 + 12 + 7 = N$

Subtracting

$N = 2310 = 2^1 \cdot 3^1 \cdot 5^1 \cdot 7^1 \cdot 11^1$

It can be resolved as product of two co-prime divisors $= 2^{n-1} = 2^{5-1} = 16$

Where '$n$' is number of distinct primes.

## SECTION 2 | MULTIPLE CORRECT ANSWERS TYPE

**5.(AB)** $\cos(\cos x - \sin x) = \cos\left(\frac{\pi}{2} - (\sin x + \cos x)\right)$

$\Rightarrow \quad \cos x - \sin x = 2n\pi \pm \left(\frac{\pi}{2} - (\sin x + \cos x)\right)$

(+) (−)

$\Rightarrow \quad \cos x = n\pi + \frac{\pi}{4} \qquad \sin x = -n\pi + \frac{\pi}{4}$

Clearly, $n = 0$ Clearly $n = 0$

$\Rightarrow \quad \cos x = \frac{\pi}{4} \quad \Rightarrow \quad \sin x = \frac{\pi}{4} \quad \Rightarrow \quad \sin x = \pm\frac{\sqrt{16-\pi^2}}{4}$

**6.(BD)** LHL $x = -h$

$$\lim_{h\to 0}\left(-\sin h + \cos h\right)^{\frac{-1}{\sin h}} = e^{\lim_{h\to 0}\left(-\sin h+\cos h-1\right)\cdot\left(\frac{-1}{\sin h}\right)} = e^{\lim_{h\to 0}\frac{(1-\cos h+\sin h)}{\sin h}} = e \ \ (LH\ Rule) \quad \therefore \quad a = e$$

*RHL*

$$\lim_{h\to 0}\frac{e^{\frac{1}{h}} + e^{\frac{2}{h}} + e^{\frac{3}{h}}}{e\, e^{2/h} + be^{3/h}} = e$$

$$\lim_{h\to 0}\frac{\frac{1}{e^{2/h}} + \frac{1}{e^{1/h}} + 1}{\frac{e}{e^{1/h}} + b} = e$$

(Dividing by $e^{3/h}$ ) $\Rightarrow \quad \frac{1}{b} = e \quad \Rightarrow \quad b = \frac{1}{e}$

**7.(ACD)**

$\lim_{x\to 0} f(x)$ DNE

LHL at $x = \pi$

$$\lim_{x\to\pi^-}\frac{\cos^{-1}\left(\cos(\sin x)\right) - |x-\pi|}{\sin^3 x} = \lim_{x\to\pi^-}\frac{\sin x + (x-\pi)}{\sin^3 x} \qquad (\because \sin x > 0 \text{ as } x \to \pi^-)$$

Let $x - \pi = t$

$$\lim_{t\to 0^-}\frac{-\sin t + t}{-\sin^3 t} = \frac{-1}{6}$$

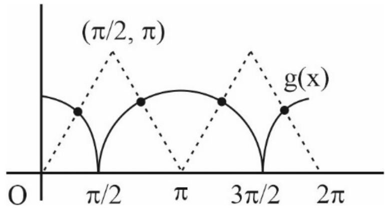
RHL at $x = \pi$

$$\lim_{x\to\pi^+}\frac{-\sin x - (x-\pi)}{\sin^3 x} \qquad (\because \sin x < 0 \text{ as } x \to \pi^+)$$

Put $x - \pi = t = -\lim_{t\to 0}\frac{\sin t + t}{\sin^3 t} = \frac{1}{6} \quad \therefore \quad$ Jump of discontinuity $= \frac{1}{6} - \left(\frac{-1}{6}\right) = \frac{1}{3}$

**8.(ABCD)** $f(x) = \cos^{-1}(\cos 2x),\ g(x) = |\cos x|$

$f(x)$, and $g(x)$ both are even and periodic so max $\{f(x), g(x)\}$ and min$\{f(x),\ g(x)\}$ will also be periodic and even.




But $\max\{f(x),\ g(x)\}$ will be non-differentiable when $f(x)=g(x)$ no of points where $f(x)=g(x)$ are four in $[0, 2\pi]$

**9.(ABCD)** $f\left(x^2+y\right)=\left(f(x)\right)^2+f(y)$

$f(0)=\left(f(0)\right)^2+f(0) \qquad f(0)=0$

for $y=0$: $f\left(x^2\right)=\left(f(x)\right)^2 \qquad \ldots$(i)

for $y=-x^2$: $0=f(0)=\left(f(x)\right)^2+f\left(-x^2\right) \quad \Rightarrow \quad \left(f(x)\right)^2=-f\left(-x^2\right) \qquad \ldots$(ii)

From (i) and (ii)

$f\left(-x^2\right)=-f\left(x^2\right)$

$\Rightarrow f(-x)=-f(x) \qquad \ldots$(iii)

Thus $f(x)$ is an odd function if $f(x)$ even also, then $f(-x)=f(x) \qquad \ldots$(iv)

$\therefore \quad f(x)=0$ for all $x \qquad$ {by (iii) and (iv)}

Since $f(x)$ is continuous at $x=0$,

$\therefore \quad \lim_{h\to 0} f(h)=0 \quad \Rightarrow \quad \lim_{h^2\to 0} f\left(x+h^2\right)=\lim_{h^2\to 0}\left\{\left(f(h)\right)^2+f(x)\right\}=f(x)$

$\Rightarrow$ it is continuous everywhere

Since $\lim_{h\to 0}\dfrac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\dfrac{f(h)}{h}$ exists

$\therefore \quad \lim_{h\to 0}\dfrac{f\left(x+h^2\right)-f(x)}{h^2}=\lim_{h\to 0}\dfrac{\left(f(h)\right)^2+f(x)-f(x)}{h^2}=\left(f'(0)\right)^2$

$\therefore \quad f'(x)=\left(f'(0)\right)^2 \Rightarrow f'(0)=\left(f'(0)\right)^2 \quad \Rightarrow \quad f'(0)=0$ or $f'(0)=1$

$\therefore \quad f(x)=\left(f'(0)\right)^2 x$

Now (i) if $f'(0)=0$, then $f(x)=0=x\,f'(0)$

(ii) if $f'(0)=1$, then $f(x)=x=xf'(0)$

**10.(BC)** $\left(2^{\frac{\pi}{\cos^{-1}x}}\right)^2-\left(a+\dfrac{1}{2}\right)2^{\frac{\pi}{\cos^{-1}x}}-a^2=0$

let $2^{\frac{\pi}{\cos^{-1}x}}=t \Rightarrow t\geq 2$

Now equation $t^2-\left(a+\dfrac{1}{2}\right)t-a^2=0$ has one root 2 or greater than 2 and other root less than 2

$\Rightarrow \quad f(2) \le 0 \quad \Rightarrow \quad 4-\left(a+\frac{1}{2}\right)2-a^2 \le 0$

$4-2a-1-a^2 \le 0$

$a^2+2a-3 \ge 0$

$(a+3)(a-1) \ge 0 \Rightarrow a \le -3 \text{ or } a \ge 1$

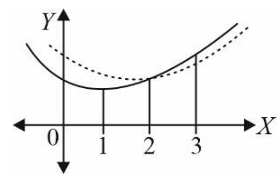
**SECTION 3 | SINGLE DIGIT INTEGER TYPE**

**1.(2)** $\frac{a+b}{2}=\frac{5}{4}\sqrt{ab} \quad \Rightarrow \quad \frac{a}{b}=\frac{1}{4}$

$\frac{H_8-a}{b-H_8}=8\times\frac{a}{b}=8\times\frac{1}{4}=2$

**2.(1)**

**3.(4)** $P(x)=\underbrace{(m^2+4m+5)}_{\text{always positive}}x^2-4x+7$




Now, abscissa of vertex $=\frac{-b}{2a}=-\left(\frac{-4}{2(m^2+4m+5)}\right)=\frac{2}{(m+2)^2+1}$

$\therefore$ Abscissa of vertex $=\frac{-b}{2a} \le 2$

Now, $3 \le x \le 5$, so

$P(x)|_{\min}=P(3)=9(m^2+4m+5)-12+7=9(m+2)^2+4$

$\therefore$ Minimum of minimum value of $P(x)$ is = 4 at $m = -2$.

**4.(3)** LH Rule

$$\lim_{x\to 0}\frac{2f'(x)-6f'(2x)+4f'(4x)}{2x}=\lim_{x\to 0}\frac{2f''(x)-12f''(2x)+16f''(4x)}{2}=\frac{2p-12p+16p}{2}=\frac{6p}{2}$$

$=3p$

**5.(3)** Equate LHS and RHS and match the coefficients.

**6.(1)** Doubt at $x = 0$ and 1

Apply *LHD*, *RHD*

At $x = 0$ *LHD* $\neq$ *RHD*

**7.(2)** $\because \quad \frac{\sin\alpha+\sin\beta+\sin\gamma}{\sin(\alpha+\beta+\gamma)}=\frac{\cos\alpha+\cos\beta+\cos\gamma}{\cos(\alpha+\beta+\gamma)}$

$$=\frac{\sin(\alpha+\beta+\gamma)(\sin\alpha+\sin\beta+\sin\gamma)+\cos(\alpha+\beta+\gamma)(\cos\alpha+\cos\beta+\cos\gamma)}{\sin(\alpha+\beta+\gamma)\sin(\alpha+\beta+\gamma)+\cos(\alpha+\beta+\gamma)\cos(\alpha+\beta+\gamma)}$$

$\left(\because \frac{a}{b}=\frac{c}{d}=\frac{xa+yc}{xb+yd}\right)$ (by ratio and proportion)

$$=\frac{\cos(\alpha+\beta)+\cos(\alpha+\gamma)+\cos(\gamma+\alpha)}{1}=2 \quad \Rightarrow \quad a=2$$

$$\therefore \quad \lim_{x\to 2}\frac{\sqrt{x^2-4}}{\sqrt{x-2}+\sqrt{x}-\sqrt{2}}=2$$

**8.(2)** $\displaystyle \lim_{x\to 0} \sum_{k=1}^{2013} \frac{\left\{\frac{x}{\tan x} + 2k\right\}}{2013} = \lim_{x\to 0}\left\{\frac{x}{\tan x}\right\}$

$\displaystyle \lim_{x\to 0}\left(\frac{x}{\tan x} - \left[\frac{x}{\tan x}\right]\right) = \lim_{x\to 0}\left(\frac{x}{\tan x}\right) = 1 \quad \therefore \quad 1^{st}\text{ term of G.P.} = 1$

$$\text{Common ratio} = \lim_{x\to 0} \frac{e^{x^3}\left(e^{\tan^3 x - x^3} - 1\right)\left(\tan^3 x - x^3\right)}{\frac{2\ln(1+x^3\sin^2 x)}{x^3\sin^2 x}\cdot x^3\sin^2 x\cdot(\tan^3 x - x^3)}$$

$$= \frac{1}{2}\lim_{x\to 0}\left(\frac{\tan x - x}{x^3}\right)\left(\frac{\tan^2 x + x^2 + x\tan x}{x^2}\right) = \frac{1}{2}\cdot\frac{1}{3}\times 3 = \frac{1}{2}$$

$$\therefore \quad S_\infty = \frac{1}{1-\left(\frac{1}{2}\right)} = 2$$